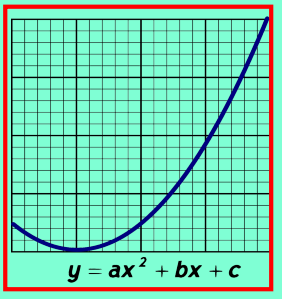
Math 125

Spring 2021

Lecture 24



If $u^2 = K$, then $u = \pm\sqrt{K}$.

Square-Root Method

Solve $x^2 = -64$

by S.R.M.

$x = \pm\sqrt{-64}$

$= \pm\sqrt{64}\sqrt{-1}$

$x = \pm 8i$

$\hookrightarrow \{\pm 8i\}$

Solve $(x+5)^2 = -24$ by Square-Root Method.

by S.R.M.

$x+5 = \pm\sqrt{-24}$

$x = -5 \pm \sqrt{4}\sqrt{6}\sqrt{-1}$ ($\sqrt{-1} \to i$)

$x = -5 \pm 2\sqrt{6}\,i$

$\{-5 \pm 2i\sqrt{6}\}$

Solve by S.R.M.: $(2x-7)^2 = -75$

$2x-7 = \pm\sqrt{-75}$

$2x-7 = \pm\sqrt{25}\sqrt{3}\sqrt{-1}$

$2x = 7 \pm 5\sqrt{3}\,i$

$x = \frac{7}{2} \pm \frac{5\sqrt{3}}{2}\,i$

$\left\{\frac{7}{2} \pm \frac{5\sqrt{3}}{2}i\right\}$

Making a Perfect-Square

$$x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$$

Ex: $x^2 + 8x + 4^2 = (x + 4)^2$

$\frac{1}{2}(8) = 4$ $\quad x^2+8x+16 = (x+4)^2$

$x^2 - 6x + (-3)^2 = (x - 3)^2$

$\frac{1}{2}\cdot(-6) = -3$ $\quad x^2 - 6x + 9 = (x-3)^2$

$x^2 + 9x + \left(\frac{9}{2}\right)^2 = \left(x + \frac{9}{2}\right)^2$

$\frac{1}{2}\cdot(9) = \frac{9}{2}$ $\quad x^2 + 9x + \frac{81}{4} = \left(x + \frac{9}{2}\right)^2$

$x^2 - 7x + \left(\frac{-7}{2}\right)^2 = \left(x - \frac{7}{2}\right)^2$

$\frac{1}{2}(-7) = \frac{-7}{2}$ $\quad x^2 - 7x + \frac{49}{4} = \left(x - \frac{7}{2}\right)^2$

$x^2 + \frac{3}{2}x + \left(\frac{3}{4}\right)^2 = \left(x + \frac{3}{4}\right)^2$

$\frac{1}{2}\cdot\frac{3}{2} = \frac{3}{4}$ $\quad x^2 + \frac{3}{2}x + \frac{9}{16} = \left(x + \frac{3}{4}\right)^2$

$x^2 - \frac{2}{5}x + \left(\frac{-1}{5}\right)^2 = \left(x - \frac{1}{5}\right)^2$

$\frac{1}{2}\cdot\frac{-2}{5} = \frac{-1}{5}$ $\quad x^2 - \frac{2}{5}x + \frac{1}{25} = \left(x - \frac{1}{5}\right)^2$

Solving by Completing the Square Method:

$x^2 - 8x + 20 = 0$

$x^2 - 8x + (-4)^2 = -20 + (-4)^2$

$\frac{1}{2} \cdot (-8) = -4$

$(x-4)^2 = -20 + 16$

$(x-4)^2 = -4$

Use S.R.M.

$x - 4 = \pm\sqrt{-4}$

$x = 4 \pm 2i$

$\{4 \pm 2i\}$

Solve by Completing the Square method:

$x^2 + 6x + 21 = 0$

$x^2 + 6x + 3^2 = -21 + 3^2$

$\frac{1}{2}(6) = 3$

$x^2 + 6x + 9 = -21 + 9$

$(x+3)^2 = -12$

Use S.R.M.

$x + 3 = \pm\sqrt{-12}$

$x = -3 \pm \sqrt{4}\sqrt{3}\sqrt{-1}$

$= -3 \pm 2\sqrt{3}\,i$

$\{-3 \pm 2\sqrt{3}i\}$

$ax^2 + bx + c = 0 \; ; a \neq 0$

Quadratic Equation

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$b^2 - 4ac$ — Discriminant

Quadratic formula

$2x^2 - 5x - 7 = 0$

$ax^2 + bx + c = 0$

$a = 2 \qquad b = -5 \qquad c = -7$

$b^2 - 4ac = (-5)^2 - 4(2)(-7)$

$= 25 + 56 = 81$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-5) \pm \sqrt{81}}{2(2)} = \frac{5 \pm 9}{4}$$

$x = \frac{5+9}{4}$

$= \frac{14}{4} = \frac{7}{2}$

$x = \frac{5-9}{4}$

$= \frac{-4}{4} = -1$

$\left\{-1, \frac{7}{2}\right\}$

Solve by Quadratic formula:

$$3x^2 + 2x - 5 = 0$$

$a = 3 \qquad b = 2 \qquad c = -5$

$$b^2 - 4ac = 2^2 - 4(3)(-5) = 64$$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-2 \pm \sqrt{64}}{2(3)} = \frac{-2 \pm 8}{6}$$

$$x = \frac{-2+8}{6} = \frac{6}{6} = \boxed{1} \qquad x = \frac{-2-8}{6} = \frac{-10}{6} = \boxed{\frac{-5}{3}}$$

$$\left\{\frac{-5}{3}, 1\right\}$$